Non-orthogonal Redundant Configurations of Single-Axis Strapped-Down Gyros

A. K. Bejczy

Guidance and Control Division

The functional reliability of the inertial reference unit of a three-axis stabilized spacecraft for long lifetime missions is of specific concern. Realizing the highly efficient redundancy potential inherent to non-orthogonal emplacements of more than three single-axis strapped-down gyros, and considering practical implementation and performance criteria, 45-deg skew-symmetric six- and four-gyro configurations based on the geometry of a regular octahedron are presented in this article. These configurations result in optimum performance, exhibit unique simplicity in gyro output processing requirements, allow high-density packaging, and require reasonable fabrication and test procedures. The redundancy potential of an octahedron six-array gyro configuration is twice as high as that of six gyros in an orthogonal emplacement. An octahedron-based skew emplacement of four gyros can provide minimum redundancy for the inertial reference unit, which, for an orthogonal emplacement, could only be obtained by six gyros.

Introduction

For three-axis stabilized spacecraft designed for long lifetime (8-12 yr) missions, the functional reliability of the inertial reference unit (IRU) constructed of single-axis strapped-down gyros is of specific concern, since the IRU has an irreplaceable functional assignment in the following critical guidance and control system operations: celestial reference acquisition, commanded turns, midcourse thrusting, planet encounters, and celestial reference occultations or disturbances in the field-of-view of the electro-optical attitude reference sensors. In addition to the critical in-line functions, the IRU can also provide a significant backup for the celestial attitude reference sensors in any phase of a mission.

Because of the very mechanism of attitude sensing through single-axis strapped-down gyros, increasing the functional reliability of the IRU must be sought not only on the component level of the single gyro instruments, but also on the instrument level of the total IRU by employing a

redundant number of gyros. The physics of attitude sensing by single-axis strapped-down gyros indicates two geometrically different ways to employ a redundant number of gyros in the IRU: (1) paralleling two or more gyros on each of the three orthogonal control axes of the spacecraft, and (2) emplacing the sensitive axes of more than three gyros in a linearly independent non-orthogonal configuration. Simple combinatorial calculations show that the redundant non-orthogonal gyro configurations are *much more efficient* in increasing the functional reliability of the IRU than are conventional redundancy techniques that simply parallel two or more gyros on the spacecraft's three orthogonal control axes.

The highly effective redundancy in full attitude reference inherent to the non-orthogonal gyro configurations is physically based on the *coupled* properties of attitude information obtainable through a linearly independent non-orthogonal emplacement of the sensitive axes of a redundant ($n > 3$) set of gyros. In such configurations, *no one gyro is restricted to replace the function of any specified gyro.* For example, out of six gyros, *any three* can fail without losing full attitude reference. This would practically result in *triple* redundancy for attitude information on each of the three orthogonal control axes of a three-axis stabilized spacecraft.

General Properties of Non-Orthogonal Gyro Configurations

For $n > 3$ single-axis strapped-down gyros in a non-orthogonal configuration, the total measurement vector in a compact vector-matrix notation is

$$\mathbf{h} = \mathbf{A}\mathbf{w} \tag{1}$$

or, in component form,

$$h_1 = A_{1x}w_x + A_{1y}w_y + A_{1z}w_z \tag{1a}$$

$$h_2 = A_{2x}w_x + A_{2y}w_y + A_{2z}w_z \tag{1b}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$h_n = A_{nx}w_x + A_{ny}w_y + A_{nz}w_z \tag{1n}$$

where

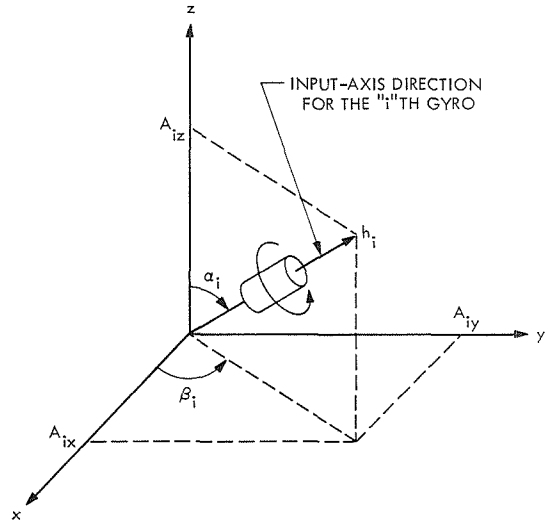
$\mathbf{h} = (h_1,\cdots,h_n)'$, an n-dimensional column vector representing the outputs of the n gyros. Depending on the mode of operation of the gyros, $\mathbf{h}$ is the angular rate or integrated rate (angular displacement) measured by the $1,2,\cdots,n$ gyros along the $1,2,\cdots,n$ gyro input-axis directions.

$\mathbf{A} = (A_{ij})$, an n-by-3 matrix of direction cosines relating the n gyro input-axis orientations to the x,y,z (pitch, yaw, roll) orthogonal control axes of the spacecraft (Figure 1).

$\mathbf{w} = (w_x,w_y,w_z)'$, a 3-dimensional column vector. Depending on the mode of operation of the gyros, $\mathbf{w}$ represents the three angular rates or integrated rates (angular displacement) along the x,y,z spacecraft control axes.[1]

The quantities of interest for attitude control are w_x,w_y,w_z, which now are not measured directly, but should be computed from Equations 1a–1n. Out of the n equations, only three are needed to determine the three "unknowns" w_x,w_y,w_z. In fact, *any group of three* linearly independent equations yield solutions for w_x,w_y,w_z. (Linear independency means that the input axes of the corresponding three gyros are not colinear or coplanar.)

[1]For rate integrating gyros, the linear relationship given by Equation 1 will accurately relate the measured angular increments h_i to the x,y,z body-axis angular increments w_x,w_y,w_z if the angular rates are low. This condition is satisfied for the intended application.



$$h_i = A_{ix}w_x + A_{iy}w_y + A_{iz}w_z$$

$$A_{ix} = \sin \alpha_i \cos \beta_i$$

$$A_{iy} = \sin \alpha_i \sin \beta_i$$

$$A_{iz} = \cos \alpha_i$$

Figure 1. Orthogonal projections of one gyro in a skew emplacement

Having a set of n linearly independent non-orthogonal gyro orientations, there are $n!/3!\,(n-3)!$ possible combinations of three out of n gyro measurements to determine w_x,w_y,w_z. That is, $n = 4$ gives 4, $n = 5$ gives 10, and $n = 6$ gives 20 different ways to determine the three quantities of interest. In contrast, an orthogonal emplacement of $n = 6$ gyros (paralleling two gyros on each of the three orthogonal control axes of the spacecraft) yields only 8 ways to determine w_x,w_y,w_z. Thus, in combinatorial terms, six gyros in a linearly independent non-orthogonal emplacement provide *2.5 times higher redundancy* for full attitude reference than six gyros in an orthogonal emplacement.

Selecting any three (h_i,h_j,h_k) out of the n gyro measurements, the computation of "equivalent" w_x,w_y,w_z measurements requires the implementation of three equations of the following general form:

$$\left.\begin{aligned} w_x &= B_{xi}h_i + B_{xj}h_j + B_{xk}h_k \\ w_y &= B_{yi}h_i + B_{yj}h_j + B_{yk}h_k \\ w_z &= B_{zi}h_i + B_{zj}h_j + B_{zk}h_k \end{aligned}\right\} \qquad (2)$$

where $B_{xi},\cdots,B_{zk}$ are elements of the inverse of the 3-by-3 $\mathbf{A}^{(ijk)}$ submatrix of $\mathbf{A}$, the elements of which are the direction cosines of the selected i,j,k gyro input-axis orientations. Clearly, each group of three gyros has its own transform matrix $\mathbf{B}^{(ijk)}$. It is noted that $B_{xi},\cdots,B_{zk}$ are *precomputed constants* determined by the fixed direction cosines $A_{ix},\cdots,A_{kz}$. The implementation of Equation 2 is depicted in Figure 2. For low angular rates and digital outputs of rate integrating gyros, Figure 2 can be interpreted as follows: Process the h_i,h_j,h_k gyro pulses as they occur and count them in the w_x,w_y,w_z adders according to the corresponding constant transform gains $B_{xi},\cdots,B_{zk}$.

Practical Criteria for Constructing Non-Orthogonal Redundant Gyro Configurations

Theoretically, for a selected number $n>3$ of single-axis strapped-down gyros, an unlimited variety of linearly independent non-orthogonal configurations is possible. Practical considerations, however, will substantially limit the possibilities of constructing appropriate configurations that also match mission requirements and spacecraft constraints. The practical construction criteria are as follows:

(1) Minimal transformation of the inherent (random and non-random) instrument errors to the computed equivalent w_x,w_y,w_z measurements. Assuming equal, independent, and normal *random* error characteristics for each gyro, the random error transformation in terms of the variance for the x axis is

$$\sigma_x^2 = (B_{xi}^2 + B_{xj}^2 + B_{xk}^2)\sigma_0^2 \qquad (3)$$

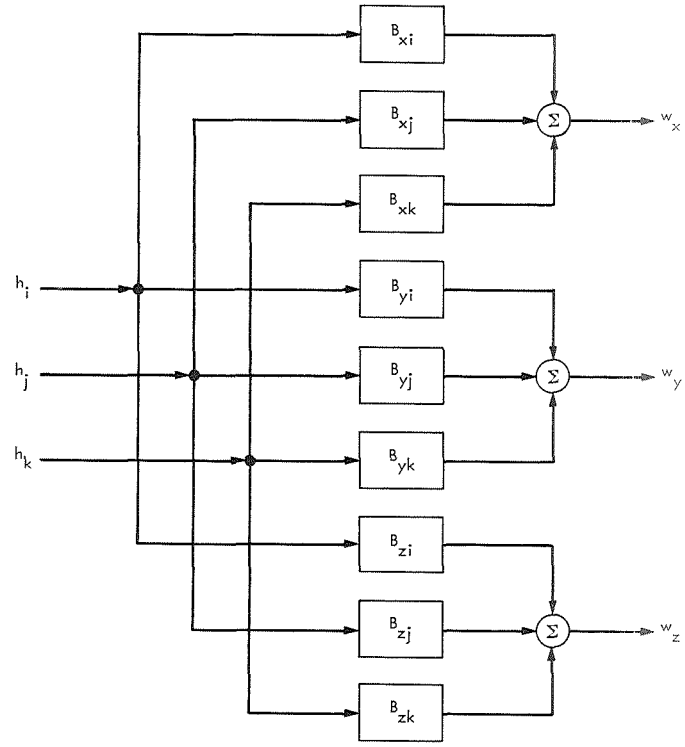


Figure 2. Mechanization scheme for measurement transformation

where σ_0^2 denotes the variance of a single gyro. The transformation of the *non-random* instrument errors (drift, misalignment) is, in the "worst case" and for the x axis, proportional to

$$|B_{xi}| + |B_{xj}| + |B_{xk}| \tag{4}$$

Clearly, similar expressions are valid for the y and z axes.

(2) Simple gyro output processing to obtain the equivalent w_x,w_y,w_z measurements. This requires some cyclic symmetry for the values of the transform gains. (For example, for six gyros in a linearly independent non-orthogonal but otherwise unrestricted emplacement, there exist *180 different* transform gain values through which to obtain equivalent w_x,w_y,w_z measurements from the 20 three-gyro combinations.)

(3) Possible compact packaging for thermal control of the gyros and economical use of spacecraft volume.

(4) Reasonable fabrication and test procedures.

References 1–5 represent the total reported works on non-orthogonal gyro configurations. With the exception of Reference 2, they mainly consider Criterion 1 and emphasize specific modes of operation or restrict

the configuration. Reference 1 considers only one four-gyro configuration and one five-gyro configuration. Reference 2 considers two orthogonal triads skewed relative to each other. References 3 and 4 describe the dodecahedron six-array configuration of the Massachusetts Institute of Technology. This work is the most elaborate and has reached the testing phase; however, its main emphasis is on a self-contained failure monitoring technique that can isolate any two failed gyros out of the six gyros, and it assumes that the IRU operation starts with six gyros turned on. Reference 5 considers mainly gyro arrays equally spaced on a 54.75-deg cone. The reported works do not emphasize Criterion 2 in any specific manner.

In this article, both Criteria 1 and 2 are basic concerns, and specific attention is paid to Criteria 3 and 4.

45-deg Emplacement Symmetry for Four and Six Gyros

A 45-deg emplacement symmetry for four and six gyros offers great advantages in processing gyro outputs to obtain the equivalent w_x,w_y,w_z measurements. Furthermore, such configurations provide optimal performance regarding instrument error transformation, require reasonable fabrication and test procedures, and can be emplaced in a high-density package.

Six-Gyro Case

Let the six gyro input-axis directions, taken one by one and identified with numbers 1 to 6, be oriented perpendicular to the opposite facing parallel edges of a regular octahedron. Let this octahedron be emplaced with its center at the origin of an x,y,z orthogonal reference system such that the octahedron's opposite facing corners are fixed to the x,y,z axes (Figure 3). The essential feature of this octahedron six-array emplacement geometry is a skew symmetry: each one of the three datum planes contains two gyro input axes aligned with 45-deg inclination to the two reference axes of the respective datum plane. Thus, each of the three reference axes obtains symmetric (45-deg) projections from four of six gyro input-axis directions. The direction cosine matrix of this configuration is then

$$A \longleftrightarrow \begin{array}{|c|c|c|c|} \hline \text{Gyro index} & x & y & z \\ \hline 1 & 0.707 & 0 & 0.707 \\ 2 & -0.707 & 0 & 0.707 \\ 3 & 0.707 & 0.707 & 0 \\ 4 & 0.707 & -0.707 & 0 \\ 5 & 0 & 0.707 & 0.707 \\ 6 & 0 & 0.707 & -0.707 \\ \hline \end{array} \qquad (5)$$

The gyro output transformations into equivalent w_x,w_y,w_z measurements are listed in Table 1 for all 16 useful three-gyro combinations. The four coplanar (hence, of no value) three-gyro combinations are also indicated. Each 3-by-3 block in Table 1 should be interpreted in a vector-matrix sense, having a mechanization scheme as depicted in general terms in Figure 2. The numbers above each block identify three gyros by their index numbers defined in Figure 3 and Equation 5. The values inside each block identify the constant transform gains relating the corresponding gyro outputs to the three (w_x,w_y,w_z) quantities of interest.

Table 1 clearly shows the unique advantage in measurement processing associated with the six-array configuration: *All gyro outputs in all three-gyro combinations have one common scaling factor, $c = 0.707$ [more precisely, $c = \cos(45\ \text{deg}) = 2^{-\frac{1}{2}}$]*. The common scaling factor of the gyro pulse signals can be considered as part of the instrument electronics of the gyros or as a designed property of the w_x,w_y,w_z adders. Thus, having scaled all gyro outputs by c, the elements in the sixteen 3-by-3 transform matrices are $0,\pm1,\pm2$. Moreover, there are many common rows and columns in the 16 transform matrices given in Table 1. For example, in

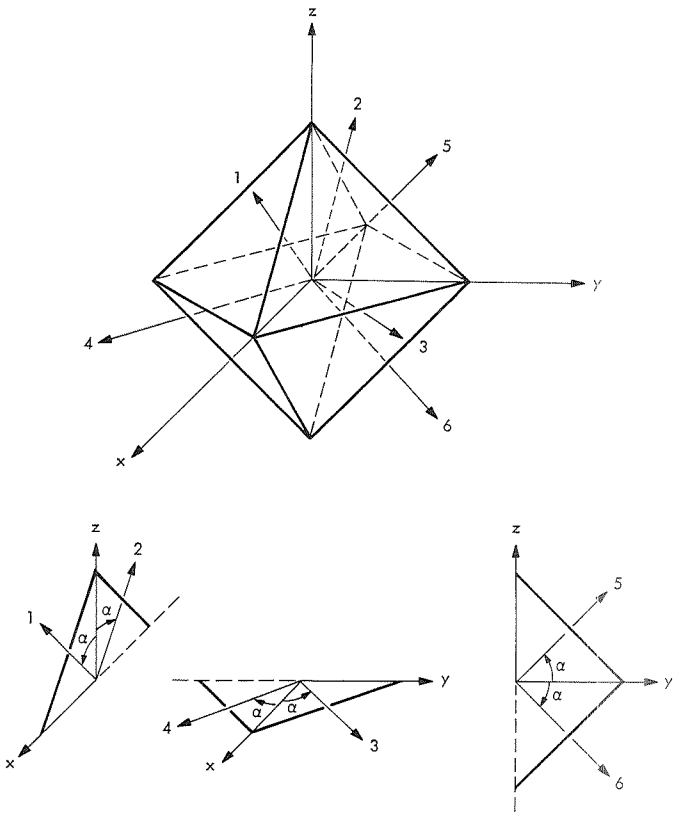


Figure 3. Octahedron six-array gyro configuration ($\alpha = 45$ deg)

Table 1. Transform gains for the three-gyro combinations of the octahedron six-array configuration (σ^2_{ir} = relative variance, c = 0.707)

	1	2	3	σ^2_{ir}	1	2	4	σ^2_{ir}	1	2	5	σ^2_{ir}	1	2	6	σ^2_{ir}
w_x	c	$-c$	0	1.0	c	$-c$	0	1.0	c	$-c$	0	1.0	c	$-c$	0	1.0
w_y	$-c$	c	$2c$	3.0	c	$-c$	$-2c$	3.0	$-c$	$-c$	$2c$	3.0	c	c	$2c$	3.0
w_z	c	c	0	1.0	c	c	0	1.0	c	c	0	1.0	c	c	0	1.0
	1	3	4	σ^2_{ir}	1	3	5	σ^2_{ir}	1	3	6	σ^2_{ir}	1	4	5	σ^2_{ir}
w_x	0	c	c	1.0	c	c	$-c$	1.5								
w_y	0	c	$-c$	1.0	$-c$	c	c	1.5	Coplanar combination				Coplanar combination			
w_z	$2c$	$-c$	$-c$	3.0	c	$-c$	c	1.5								
	1	4	6	σ^2_{ir}	1	5	6	σ^2_{ir}	2	3	4	σ^2_{ir}	2	3	5	σ^2_{ir}
w_x	c	c	c	1.5	$2c$	$-c$	c	3.0	0	c	c	1.0				
w_y	c	$-c$	c	1.5	0	c	c	1.0	0	c	$-c$	1.0	Coplanar combination			
w_z	c	$-c$	$-c$	1.5	0	c	$-c$	1.0	$2c$	c	c	3.0				
	2	3	6	σ^2_{ir}	2	4	5	σ^2_{ir}	2	4	6	σ^2_{ir}	2	5	6	σ^2_{ir}
w_x	$-c$	c	$-c$	1.5	$-c$	c	c	1.5					$-2c$	c	$-c$	3.0
w_y	c	c	c	1.5	$-c$	$-c$	c	1.5	Coplanar combination				0	c	c	1.0
w_z	c	c	$-c$	1.5	c	c	c	1.5					0	c	$-c$	1.0
	3	4	5	σ^2_{ir}	3	4	6	σ^2_{ir}	3	5	6	σ^2_{ir}	4	5	6	σ^2_{ir}
w_x	c	c	0	1.0	c	c	0	1.0	$2c$	$-c$	$-c$	3.0	$2c$	c	c	3.0
w_y	c	$-c$	0	1.0	c	$-c$	0	1.0	0	c	c	1.0	0	c	c	1.0
w_z	$-c$	c	$2c$	3.0	c	$-c$	$-2c$	3.0	0	c	$-c$	1.0	0	c	$-c$	1.0

the 1,2,3; 1,2,4; 1,2,5; and 1,2,6 combinations, the transform gains for w_x and w_z are unchanged, and only the polarities of the (numerically unchanged) gains for w_y are varying. Clearly, these structural features of the transform matrices imply very simple implementation.

The relative variances per reference axis ($\sigma^2_{ir} = \sigma^2_i/\sigma^2_0$) due to the error transformation of the skew geometry are also listed in Table 1 for each three-gyro combination. As may be seen, in most of the cases there is no error amplification; when error amplification does exist, the factor is insignificantly small (1.5 and 3). Furthermore, even the worst-case non-random error amplification factor is reasonably small, having a value 2.82. It is interesting to note that, for the three-gyro combinations of the dodecahedron configuration (References 3 and 4), the highest relative variance is 8.35, and the worst-case non-random error amplification factor is 5. Thus, *the octahedron six-array configuration tends to minimize the maximum error amplification inherent to a skew gyro emplacement.*

The six-array configuration also allows a high-density packaging, as shown in Figure 4. Moreover, *there is no difference between the packag-*

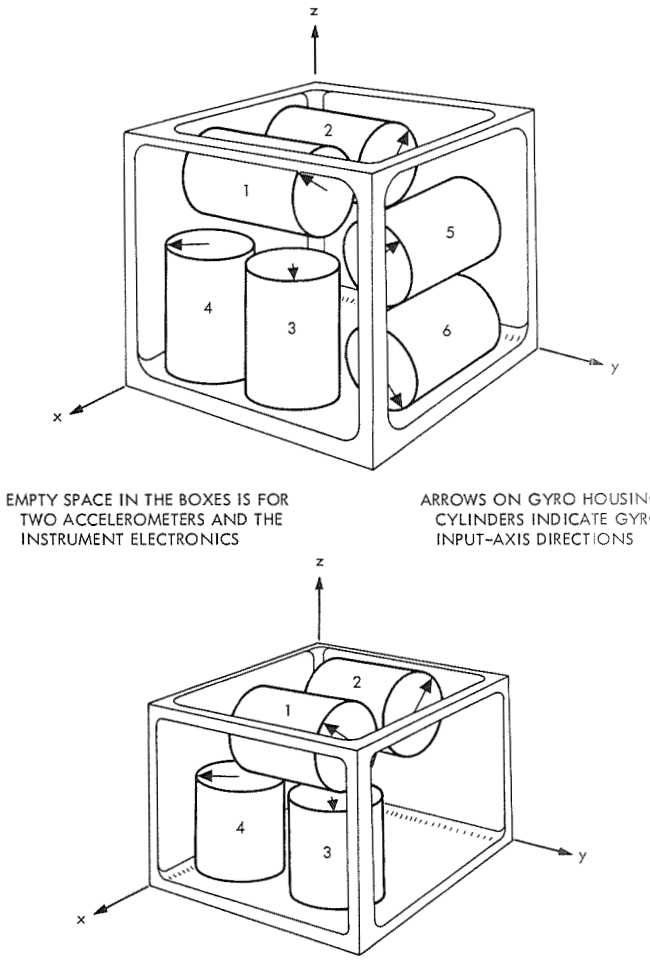


Figure 4. Packaging of six and four gyros in an octahedron-based emplacement

ing for the octahedron six-array configuration and that for an orthogonal emplacement of six gyros, as is apparent from Figure 4. Furthermore, the 45-deg emplacement symmetry implies substantial advantages in the fabrication process (making alignment fixtures, cutting angles in a metal) and in laboratory test procedures. In fact, system testing may be performed on a single-axis test pad basis.

Four-Gyro Case

When only minimum redundancy is required for the IRU, it can be provided by using four gyros in a non-orthogonal emplacement. (In an orthogonal emplacement, minimum redundancy can only be obtained by using six gyros.) Three different non-orthogonal emplacements of four gyros can very easily be constructed from the octahedron six-array configuration, *retaining all the advantages of that configuration.* The three

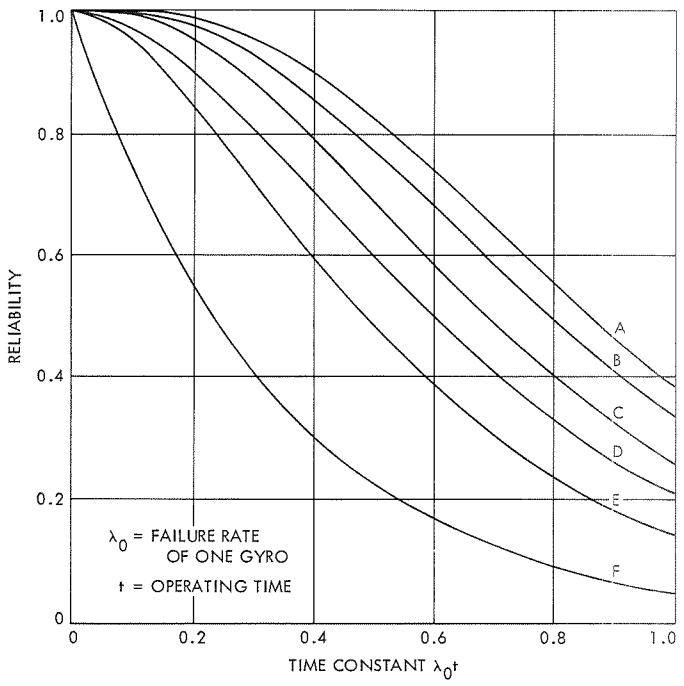
different non-orthogonal emplacements of four gyros will be equivalent to having gyros 1,2,3,4; 1,2,5,6; or 3,4,5,6 (Figure 3) in the IRU package. The transform gains corresponding to the relevant three-gyro outputs of those three four-gyro packages can easily be identified in Table 1. As may be seen, the only difference between the three four-gyro packages is that each package has a different preference axis where no error amplification occurs in all three-gyro combinations. It is interesting to note that the theoretically optimum four-gyro emplacement (Reference 1) yields a relative variance of 1.5 for all three reference axes in all three-gyro combinations, while, in the octahedron six-array-based four-gyro emplacement, the three-gyro combinations yield no error amplification on two reference axes and an error amplification factor of 3 on the third axis. This factor shifts its location between two axes for the different three-gyro combinations. A compact packaging arrangement for the octahedron six-array-based four-gyro emplacement is shown in Figure 4.

Reliability Merits

In the octahedron six-array configuration, the useful "three-out-of-six" gyro combinations for full attitude reference are reduced to 16 from 20, the maximum value achievable by a six-gyro skew configuration. It is felt, however, that the small reduction of the redundancy capability in the six-array configuration is compensated by the great simplicity in measurement processing. It is also noted that *the redundancy capability of the octahedron six-array configuration in combinatorial terms is still twice as high as the redundancy capability of six gyros in an orthogonal emplacement.*

The four coplanar three-gyro combinations of the octahedron six-array configuration have a small impact on the functional reliability of the octahedron six-array IRU, as may be seen in Figure 5, which displays the reliability functions of six different gyro configurations. Figure 5 was computed assuming perfect failure monitoring accomplished by external means (e.g., voltage sensing) and continuous operation of all gyros, with only failed gyros turned off. These assumptions have no significant effect on the *relative* reliability merits of different gyro configurations, provided: (1) failure monitoring has considerably higher reliability than that for the individual gyros, and (2) the shelf-life degradation of gyros is considerably smaller than the gyros' active wear-out rate. Additional reliability functions for different gyro configurations and operation modes and for different failure monitoring techniques can be found in Reference 6.

It is noted that the octahedron six-array-based four-gyro configurations possess the full redundancy capability that is available with a linearly independent four-gyro skew emplacement.



A OPTIMUM (SKEW) CONFIGURATION OF SIX GYROS
B OCTAHEDRON (SKEW) CONFIGURATION OF SIX GYROS
C SKEW CONFIGURATION OF FIVE GYROS
D ORTHOGONAL EMPLACEMENT OF TWO GYROS ON EACH AXIS
E SKEW CONFIGURATION OF FOUR GYROS
F NON-REDUNDANT IRU

Figure 5. Reliability functions of six gyro configurations

Concluding Remarks

Non-orthogonal redundant gyro configurations offer several useful operation options: (1) Operating four gyros at a given time, *instant* full attitude reference is available if *any one* of the four gyros has failed; (2) operating five gyros at a given time, one failed gyro can be identified by a data processing technique; and (3) even one skewed gyro is useful for in-line estimation of full attitude reference.

References

1. Barnhill, D. H., and Susens, D. C., "Digital Attitude Reference, Redundancy and Temperature Control Considerations," paper presented at the AIAA/JACC Guidance and Control Conference, Aug. 15–17, 1966.

2. Weiss, R., and Nathan, I., "An Ultra-Reliable Attitude Reference System for a Manned Orbiting Laboratory with Limited Sparing Capability," paper presented at the AIAA Guidance and Control Conference, Seattle, Wash., 1966.

References (contd)

3. Gilmore, J. P., *A Non-Orthogonal Gyro Configuration,* M.S. thesis. Massachusetts Institute of Technology, Cambridge, Jan. 1967.

4. Gilmore, J. P., and McKern, R. A., "A Redundant Strapdown Inertial System Mechanization—SIRU, paper presented at the AIAA Guidance and Control Conference, Santa Barbara, Calif., Aug. 17–19, 1970.

5. Pejsa, A. J., "Optimum Orientation and Accuracy of Redundant Sensor Arrays," paper presented at the AIAA 9th Aerospace Sciences Meeting, New York, N.Y., Jan. 25–27, 1971.

6. Bejczy, A. K., *Reliability Merits of Different Redundant Orthogonal and Non-Orthogonal Single-Axis Gyro Configurations,* JPL Internal Document EM 344-301-AKB. Jet Propulsion Laboratory, Pasadena, Calif., Jan. 21, 1971.